HBT small signal T and π model extraction using a simple, robust and fully analytical procedure

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Abstract — **A simple and fully analytical procedure is proposed in this paper for the direct extraction of the HBT small signal equivalent circuit model, whether it be the T- or hybrid π-topology, from DC and single-frequency s-parameter measurements. DC data is required since it is possible to show that direct extraction from single frequency s-parameters is not possible, without prior knowledge of one parameter. The extraction procedure developed is based on the prior knowledge of dc current gain, β_0. Excellent results have been obtained when applied to InGaP/GaAs and InP based HBTs.**

I. Introduction

In recent years a number of researchers have attempted to develop accurate small signal HBT model extraction methods. A general observation is that direct extraction of intrinsic model parameters from single frequency s-parameters has proved to be very difficult and most of them have had to combine analytical calculations, taking into account approximations and constraints, with optimization procedures [1]-[3]. The fully analytical methods also have had to make use of additional information: frequency approximations [4]-[5] previous knowledge (e.g. from special test structures or extra dc measurements) of some parameters (R_{be}, parasitic elements, etc.) [6]-[7] or involve multibias intrinsic extractions [8]. Analysis of all the relevant HBT small signal model topologies will show that the cause of these problems is the observation that direct extraction from single frequency s-parameters of any HBT model topology is not possible, without prior knowledge of one parameter.

II. Small signal HBT equivalent circuits

Fig. 1a shows a schematic of the small-signal HBT equivalent circuit. For the intrinsic block there are different choices that are summarized through figs.1b, 1c and 1d. Fig. 1e shows also two simple feedback networks that could be used in each of the preceding intrinsic circuits. Hence, there are six possible intrinsic circuit topologies.

The most common intrinsic topologies found in literature are: π-g_m (fig. 1b) and T-α (fig. 1d), both including the π-type feedback network (fig. 1e down). A third choice, less common than the preceding ones, is π-β (fig. 1c). This topology is used, for example, in [9], with a more complex feedback network than those shown in fig.1e. In this paper, we will use this last topology in conjunction with the T-type feedback network (fig. 1e up), since it highlights the extraction problem and allows for a simple analytical direct extraction formulation.

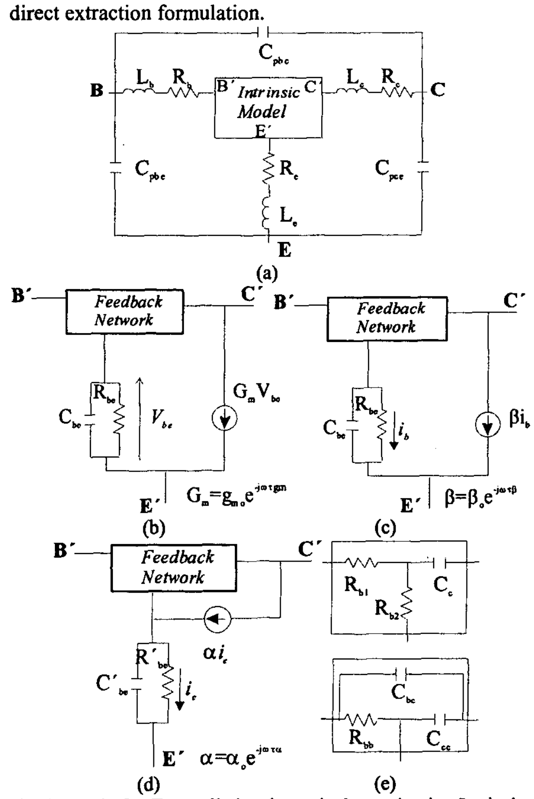

Fig. 1. (a) HBT small-signal equivalent circuit. Intrinsic topologies: (b) π-g_m (c)π-β (d) T-α. (e) Feedback networks.

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Many authors have found that over the usable frequency range of the HBT any of these three intrinsic circuit topologies are able to provide a good prediction of the measured small-signal s-parameters. In fact, they can all be related mathematically. Consider the simple transformation from intrinsic π-g_m and π-β topologies. The current sources are simply related by eq. (1)

$$\beta = G_m . R_{be} \qquad (1)$$

hence, $\beta_o = g_{mo}.R_{be}$ and $\tau_\beta = \tau_{gm}$; $\beta' = \dfrac{\beta}{1 + j\omega R_{be} C_{be}}$

These two circuits are equivalent. Similar expressions relating the parameters for the intrinsic π-gm and T-α topologies can be found in [10]. These expressions show, provided we can assume that $\omega\tau_{gm} << 1$, that all the circuit topologies are equally applicable with frequency independent element values.

Note the feedback network topology did not affect these mathematical transformations; hence consideration of this circuit topology can be accounted for separately. T-α and π-g_m topologies commonly found in the literature have the π-type feedback networks while we will use the T-type feedback network for our extraction of the π-β topology. To first order, both feedback networks are equivalent, and the transformation between them can be done by using

$$C_c \cong C_{bc} + C_{cc} \quad R_{b1} \cong \frac{C_{cc} R_{bb}}{C_c} \quad R_{b2} \cong \frac{C_{bc} R_{bb}}{C_c} \qquad (2)$$

This simplification is valid provided $\omega\tau_{bc}^2 << C_{cc}$ and $\omega\tau_{bc}^2 << C_{bc}$, where $\tau_{bc} = R_{bb}.(C_{bc}C_{cc} /(C_{bc} + C_{cc}))$. For simplification purposes, we are omitting in all this analysis the collector conductance (g_c), in parallel with C_c, and other possible additional elements.

Because these topologies are thus equivalent over a broad frequency range, they all share the same extraction limitations. The problem of finding a direct extraction procedure can be easily clarified if we consider the π-β with T-feedback topology (figs. 1c,e up). Analysis of this circuit shows that while it only has seven circuit elements, it still cannot be directly extracted from single frequency s-parameter measurements; even thought that provides us with eight parameters. The problem is that three circuit elements of the model are associated with one branch of the circuit, R_{b2}, C_{be} and R_{be}, hence, single frequency extraction of all elements in this branch is thus impossible without prior knowledge of one of the element values. This limitation must extend to the other topologies by way of the previous discussed equivalence. Hence, one parameter must always be derived independently, for example, from additional DC measurements, or analyzing the frequency dependence of some circuit component. In most cases, it is the frequency response that has been used [4]-[5]. In [7], however, they found that by using prior knowledge of the value of R_{be}, determined from DC measurements, a direct extraction was possible. An alternative would be to use the value of dc current gain β_0, which can be more easily determined from dc measurements. With this in mind, we developed a direct analytical extraction procedure based on the prior knowledge of the current gain β_0. Simply, this parameter can be determined from the dc bias currents flowing when measuring the s-parameters, however, more robust methods are more accurate. One could argue that similarly, we could start our extraction from the knowledge of the dc value of α_0, and this is true. In fact, it has not been uncommon in other extractions coupled with optimization to fix the value of α_0 to its dc value to help in the optimization procedure [3].

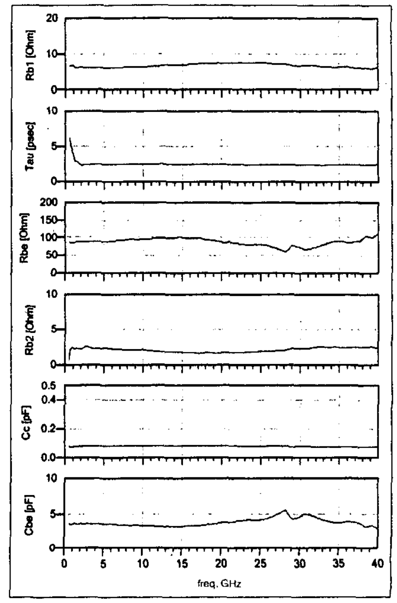

Fig. 2. Celeritek HBT-1 2x3x27 μm^2. Frequency range: 0.5-40 GHz. Bias point: Ib=0.3 mA, Vce=2 V. Extracted circuit parameters vs. frequency for the π-β (with T- feedback) model.

III. Model Analytical Extraction

The approach taken in this paper is to take advantage of the π-β HBT topology and T-type feedback topology and prior knowledge of β_0 to extract the model element values in a very simple and direct manner. Afterwards, we can transform the obtained parameters to any other desired topologies, typically the π-α or π-g_m with π-type feedback.

Cpbe	Cpbc	Cpce	Lb	Lc	Le	Rb
30fF	0	30fF	15pH	5pH	7.5pH	0
Rc	**Re**	**Rb1**	**Rb2**	**Cc**	**Rbe**	**Cbe**
1Ω	.1Ω	6.79Ω	1.85Ω	81fF	95.2Ω	3.4pF
τβ	**Ccc**	**Cbc**	**Gmo**	**Rbb**	**βo**	
2.4ps	63fF	17fF	513mS	8.6Ω	48	

Table I. HBT-1 2x3x27 μm^2. Frequency: 0.5-40 GHz. Bias: Ib=0.3 mA, Vce=2 V. π-β and π–g_m model parameters.

Initially, the parasitic values are obtained from s-parameter HBT measurements under reverse and highly forward bias conditions, as in [9]. Problems associated with parasitic extraction are common to all model extraction techniques, especially when the information is provided by a single device (different geometries or text structures are not available). Next step is to measure the device in active bias conditions over a wide frequency range. Using conventional matrix transformations (s to y, y to z and z to y) we de-embed the measured s-parameters from the parasitic cells to obtain the intrinsic Z_{ij} parameters.

β_0 can be obtained simply from the bias information ($\beta_0=I_c/I_b$). This value, in conjunction with the intrinsic Z_{ij} parameters, will be used to directly extract the rest of the intrinsic model parameters. We also have a frequency dependent approach to extract β_0 that is valid provided that we have a good previous estimation of the parasitic parameters. The use of an extraction based on dc β_0 is hopefully not affected by this restriction, but is influenced by thermal effects. However, we have found that the ability of the extracted model to predict accurately the measured s-parameters is not dependent on which β_0 *extraction procedure is used.*

The model parameters can be equated taking into account some useful definitions:

$$\tau_{be} = R_{be}C_{be} \qquad Z_e = R_{b2} + \frac{R_{be}}{1+j\omega\tau_{be}}$$

$$\beta' = \frac{\beta_0 e^{-j\omega\tau_\beta}}{1+j\omega\tau_{be}} \qquad Z_c = \frac{1}{j\omega C_c}$$

The intrinsic device Z_{ij} parameters are now determined as a function of these parameters;

$$Z_{11} = R_{b1} + \frac{Z_e}{1+\beta'} \quad Z_{12} = \frac{Z_e}{1+\beta'}$$

$$Z_{21} = \frac{Z_e - \beta' Z_c}{1+\beta'} \quad Z_{22} = \frac{Z_e + Z_c}{1+\beta'}$$

From these relations, the extraction procedure is as follows

$$\beta' = \frac{Z_{12}-Z_{21}}{Z_{22}-Z_{12}} \qquad R_{b1} = \mathrm{real}(Z_{11}-Z_{12})$$

$$Z_e = Z_{12}\left(\frac{Z_{22}-Z_{21}}{Z_{22}-Z_{12}}\right) \qquad Z_c = Z_{22}-Z_{21}$$

$$\tau_{be} = \sqrt{\frac{\left(\frac{\beta_0^2}{|\beta'|^2}-1\right)}{\omega^2}} \qquad \tau_\beta = \frac{-\mathrm{phase}(\beta'(1+j\omega\tau_{be}))}{\omega}$$

$$R_{be} = \frac{-\mathrm{imag}(Z_e)(1+\omega^2\tau_{be}^2)}{\omega\tau_{be}} \qquad R_{b2} = \mathrm{real}(Z_e) - \frac{R_{be}}{1+\omega^2\tau_{be}^2}$$

$$C_{be} = \frac{\tau_{be}}{R_{be}} \qquad C_c = \frac{-1}{\mathrm{imag}(Z_c)\omega}$$

As can be seen, if β_0 is known we can directly extract at each frequency all the intrinsic model parameters. If the initial values of the parasitic elements are accurate, we obtain intrinsic parameter values that are frequency independent. If this is not the case, we could always use the variance over frequency of each parameter to further optimize the parasitic values. Once this problem is solved, and to eliminate noise in the measurements affecting extracted parameter values at each frequency, we could apply a robust estimator like the median to compute, from all the parameter values in the measured frequency range, a unique parameter value. Conversion from the T-type feedback to the π-type feedback to get the more commonly used circuit topology involves a simple transformation using equations (2). Finally we can use the transformation $g_{mo} = \beta_0 / R_{be}$ if required.

IV. Results

We have applied this extraction procedure to InGaP/GaAs and InP based HBT devices from different wafers and device geometries. S-parameter measurements have been performed in the range 250 MHz up to 40 GHz using an on-wafer 50 Ω small signal measurement system based on the HP8510C network analyzer.

Fig. 2 shows the results of the extraction procedure applied to a 2x3x27 μm^2 InGaP/GaAs HBT device from Celeritek. The bias point is in the active mode (I_b=0.3 mA, V_{ce}=2 V). As can be seen, for the particular values of the parasitics shown in table I, the intrinsic values are practically frequency independent. In figs. 3a and 4, the comparison between measured s-parameters and those obtained with the extracted model is shown. Close agreement is obtained for both the T-and π–type feedback topologies for π-β and π–g_m, respectively (fig. 3b).

This simple extraction procedure works quite well with all the devices to which it was applied, even when we parted from very poor estimations of the parasitic values.

V. CONCLUSION

In this paper a simple and fully analytical procedure has been developed for the extraction of the HBT small signal model from conventional dc and small signal measurements. The extraction procedure is direct, independent of topologies (T– π), and is based in the prior knowledge of a robust parameter, β_0. β_0 can be simply derived from dc measurements. The robustness of this parameter can overcome extraction problems when parasitic values are difficult to get accurately prior to intrinsic extraction. It was also demonstrated the equivalence of the topologies, and the good behaviour of the resulting models when the proposed extraction strategy is applied. It must be noted that the bias dependence of extracted small signal model parameters can be directly related to large signal model formulations.

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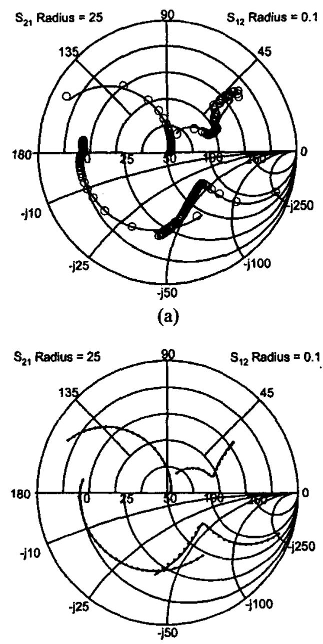

(b)

Fig. 3. HBT-1 2x3x27 μm^2. Frequency: 0.5-40 GHz. Bias: Ib=0.3 mA, Vce=2 V. (a) Measurements (circles) and simulation (line). (b) Simulations T-feedback with π-β (line) and π–feedback with π–g_m (dot).

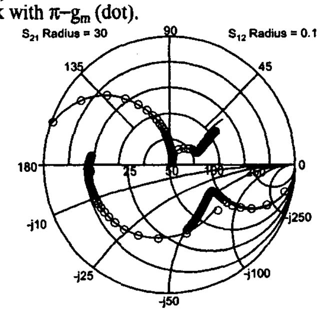

Fig. 4. HBT-2 2x3x27 μm^2. Frequency: 0.25-26.5 GHz. Bias: Ib=0.24 mA, Vce=3.4 V. (a) Measurements (circles) and simulation (π-β,line).